Project 2

Nathan Holt, Emily Kiesel, Nicole Rosato

April 4, 2017

1 Introduction

In New Mexico, and much of the south western United States, wild hogs (Sus Scrofa) are wreaking havoc. Often weighing two to three hundred pounds, the hogs have quite an appetite and will eat just about anything, including corn, sea turtle eggs, baby deer, goats, and even humans! In their quest for food, these "opportunistic omnivores" cause damage to crops, land, and property, costing around 1.5 billion dollars in damage per year [2]. It is of interest to eliminate the approximately 6 million wild hogs as they are so destructive. This has been difficult, though, because the hogs reproduce quickly, move around frequently, and are smart enough to avoid traps. Female hogs begin reproducing at 6 months of age, and can have as many as 12 babies a year [2].

Luckily, wild hogs are very messy animals that love to drink water and roll around in mud. When they do this, they leave behind traces of their DNA, called eDNA (environmental DNA). Scientists at the National Wildlife Research Center have developed a method for identifying whether a wild hog has been in the water recently based on testing for the eDNA in water samples. These samples are collected by a team from the US Department of Agriculture's Animal and Plant Health Inspection Service and shipped to the National Wildlife Research Center. If the sample tests positive for hog eDNA, Brian Achuleta from the US Department of Agriculture's Animal and Plant Health Inspection Service has his team of sharpshooters narrow their search for the wild hogs to a 10 square mile area. This has shown some results- at one site, the sharpshooter team was able to kill 8 hogs, and at another site they were able to kill 13. Archuleta's goal is to use this process to eliminate all the wild hogs by September 2017[2].

We want to determine whether the water sampling and sharpshooter strategy will be able to successfully accomplish Achuleta's goal of eliminating all the wild hogs in New Mexico by September 2017. In order to do this, we developed a model for the wild hog population, to help us examine how the population would change over time. To adequately describe how th population changes, our model accounts for the birth and natural death rate of hogs, along with the rate at which the hogs are killed off.

2 Presentation of the Model

We developed the following discrete model for the hog population.

$$P_n = P_{n-1} + \frac{1}{5}b \cdot \frac{P_{n-1}}{2} - (k + d) \cdot P_{n-1}, \quad n \geq 1 \tag{1}$$

Quantity	Meaning	Dimension	Units
P_n	Number of hogs in New Mexico on day n	Hogs	Hogs
b	Birthrate	$Hogs \cdot Hogs^{-}1$= dimensionless	unitless
d	Natural Deathrate	$Hogs \cdot Hogs^{-}1$= dimensionless	unitless
k	Kill Rate	$Hogs \cdot Hogs^{-}1$= dimensionless	unitless

The variable P_n is the population of wild hogs in New Mexico on the previous day, plus hogs that were born (and only $\frac{1}{5}$ survive the first 6 months of life), minus the hogs that died either of natural causes or from being killed by Archuleta's team.

2.1 Assumptions

In developing the model, we made a few assumptions. The main model assumption we made is that the carrying capacity of New Mexico is high enough to ignore. We felt comfortable making this assumption because the population of hogs has continually grown with little sign of stopping, and the hogs eat almost anything, so there is not a lack of resources for them. The next assumption we made was that the number of hogs migrating into New Mexico is equal to the number of hogs migrating out of New Mexico. Therefore, new hogs enter the system only through birth. We also assumed that there were no changes to be made to Archuleta's strategy (i.e. number of helicopters and shooters is the same each day, no cost issues cause anything to change). Also, we assumed that an equal number of sites come back positive with hog eDNA for each day. Lastly, we chose to ignore the costs involved with the process. The hogs are a big problem, so if New Mexico is serious about eliminating the population, they will pay as necessary.

2.2 Strategy

In order to see if it is plausible for Archuleta's strategy to successfully eliminate the hogs by September 2017, we will see how many days it takes for there to be less than one hog left. In other words, we will determine the value of n such that $P_n < 1$. Additionally, we will determine the kill rate needed in order to be successful in ever eliminating the hog population in New Mexico, as well as eliminating the hog population in New Mexico in 180 days.

3 Model Solutions

Since all of the terms in equation (1) contain P_{n-1}, we can rewrite the model as:

$$P_n = (1 + b - d - k)P_{n-1}.$$

This is a geometric sequence with comon ratio $(1 + b - d - k)$ so we can write the closed form of the model as:

$$P_n = P_0(1 + \frac{1}{10}b - d - k)^n. \tag{2}$$

3.1 Qualitative features

Analyzing the model reveals some qualitative behavior of the hog population. Since the model is a geometric sequence, the population of hogs is always positive. Also, as n goes to infinity, the hog population has three possible options.

- If $(1 + \frac{1}{10}b - d - k) > 1$, $\lim_{n\to\infty} P_n = \infty$, so the hog population would continue to grow indefinitely.

- If $(1+\frac{1}{10}b-d-k) = 1$, $\lim_{n\to\infty} P_n = P_0$, so the population would remain constant forever.

- If $(1 + \frac{1}{10}b - d - k) < 1$, $\lim_{n\to\infty} P_n = 0$, so the population would die out.

So, we know that in order for there to be a chance of eliminating the hog population, we need $(1 + \frac{1}{10}b - d - k) < 1$ or more simply $\frac{1}{10}b < d + k$. This means that the death rate must be greater than the birth rate.

3.2 Parameter Values

To determine the parameter values, we made some additional assumptions.

- Initial Population P_0:

  We assumed that there were 1 million hogs in New Mexico, since there are an estimated 2,600,000 hogs in Texas, and Texas is a little more than twice as large as New Mexico [3].

- Birth Rate:

  To determine the birthrate, we assumed that each year, the age appropriate females (6 months or older) give birth to 10 babies each, so each female gives birth to $\frac{10}{365}$ babies per day, and therefore $b = 0.0274$.

- Death Rate:

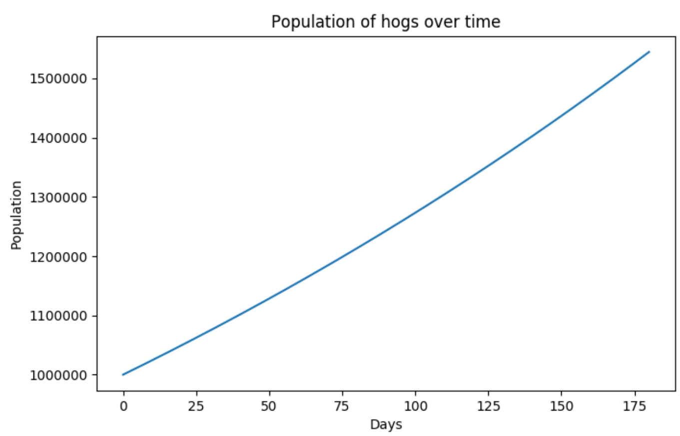
  To determine the natural death rate of wild hogs, we looked at data regarding mortality rates[1]. By estimating the slope of a survival versus time graph, we estimated the death rate of hogs to be $\frac{2}{19}$ hogs per year, or $\frac{2}{6935}$ hogs per day. Therefore, we let $d = 0.00029$.

- Kill Rate:

  To determine the rate of death due to Archuleta's team, we looked at the information provided by NPR[2] regarding the number of hogs previously killed, and New Atlas[4] regarding helicopter capabilities. We estimated that the team can reasonably kill hogs at 3 sites per day, and that when the population of hogs was 1 million, the team killed 11 hogs per site on average. Therefore, we estimated that $\frac{33}{1000000}$ of the population of hogs is killed off each day, and chose $k = .000033$.

4 Numerical Results

Using the parameter values described above, we graphed the population of the hogs over the next 6 months (180 days) as shown below.

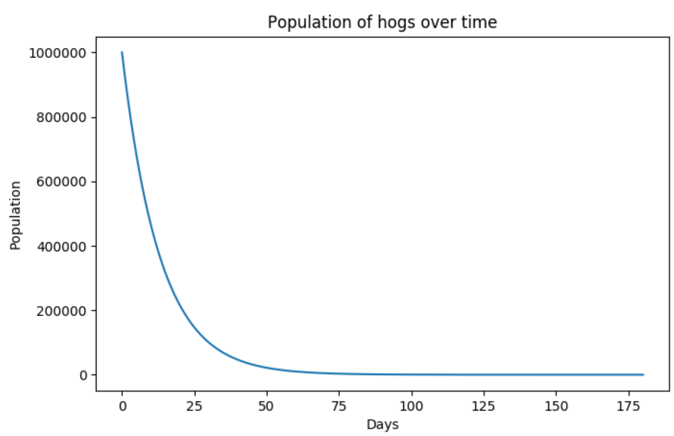


Clearly, the model suggests that even with Archuleta's team eliminating some hogs, the hog population will continue to grow. At this rate, 1 helicopter going to 3 sites each day, Archuleta's team will not be able to eliminate the hog population, let alone by September 2017.

4.1 Critical Kill Rate

In order for the population of hogs to decrease over time, we need $(1 + \frac{1}{10}b - d - k) < 1$ or $k > \frac{1}{10}b - d$. So for $b = 0.0274$ and $d = 0.00029$, if $k < .00245$ the population will eventually be eliminated. Therefore, if Archuleta's team can kill over .245% of the population each day, they can cause the hog population to decrease. If they kill .24514%, on day 13,815,504 the hog population would be less than one. This is 37,851 years. This would require killing hogs initially at 222 sites a day, which would require an estimated 74 helicopters.

However in order to kill off the population in 180 days, as defined by there being less than 1 hog left, the kill rate, k, must satisfy $P_0(1 + \frac{1}{10}b - d - k)^180 < 1$. For our values of P_0, b, and d, k must be at least 0.0763, meaning Archuleta's team must kill off 7.63% of the population each day. Below is a plot of the population over the first 175 days when $k = .0763$.



Clearly, for the kill rate of 7.63%, the hog population would decrease dramatically. This strategy would require successfully killing hogs at 6936 sites initially, requiring 2312 helicopters. This is considerably higher than what we believe Archuleta's team is currently doing, and is probably not feasible.

5 Limitations

There are some limitations of our model. Since we averaged the natural death rates across all age groups, our model doesn't account for the variance in death rate per age group. We also allow newborn pigs to be included in the population that is giving birth, which could inflate the population numbers. Additionally, it does not consider that some areas of New Mexico, such as farm land, may have a more dense population of hogs than big cities. Lastly, we ignored any time delay in collecting water samples, testing, receiving results, and locating the hogs.

6 Conclusions

In summary, the eDNA method is an inadequate method for eliminating the hog population in New Mexico without dramatically increasing the amount of resources used. The birthrate of the hogs is too high to counteract by killing less than .245% of the population everyday. If Archuleta's team continues at its current rate, it can be expected that the wild hog population will continue to increase and cause problems in New Mexico.

References

[1] Feral hog population biology, 2012.

[2] Rae Ellen Bichell. Scientists get down and dirty with dna to track wild pigs. *NPR*, 2017.

[3] Jared B. Timmons, Dr. Billy Higgenbothom, Dr. Roel Lopex, Dr. James C. Cathey, Janell Mellish, Jonathan Griffen, Dr. Aaron Sumrall, and Kevin Skow. Feral hog population growth, density and harvest in texas. *Texas A&M*, 2012.

[4] C.C. Weiss. Recco streamlines helicopter search and rescue. *New Atlas*, 2015.